10-6 Volume of Prisms and Cylinders

Warm Up

Find the area of each figure. Round to the nearest tenth.

1. an equilateral triangle with edge length 20 cm 173.2 cm^2

2. a regular hexagon with edge length 14 m 509.2 m^2

3. a circle with radius 6.8 in. 145.3 in^2

4. a circle with diameter 14 ft 153.9 ft^2

Objectives

Learn and apply the formula for the volume of a prism.

Learn and apply the formula for the volume of a cylinder.

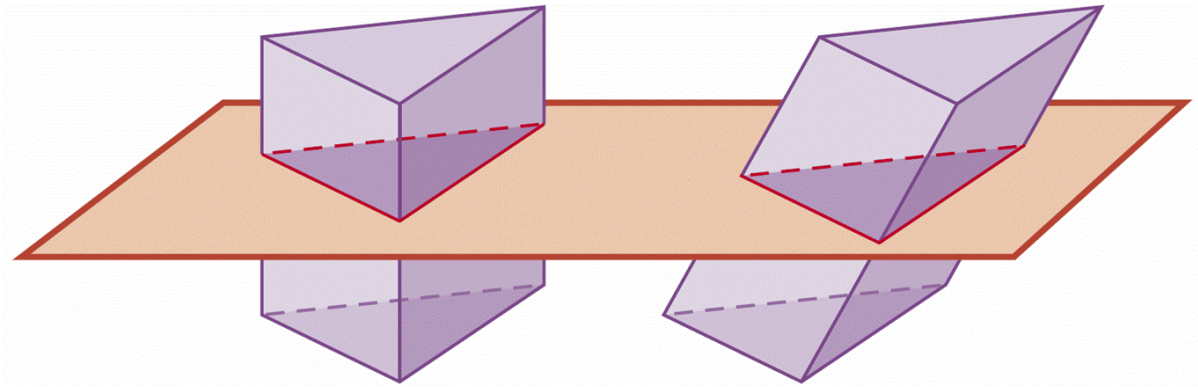
Vocabulary

volume

10-6 Volume of Prisms and Cylinders

The **volume** of a three-dimensional figure is the number of nonoverlapping unit cubes of a given size that will exactly fill the interior.

Cavalieri's principle says that if two three-dimensional figures have the same height and have the same cross-sectional area at every level, they have the same volume.

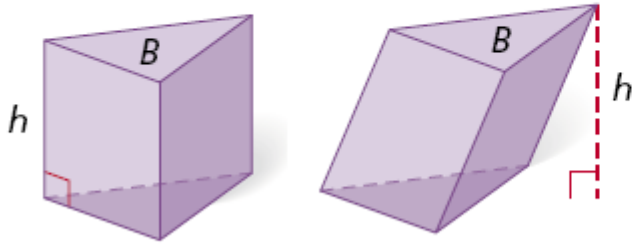


A right prism and an oblique prism with the same base and height have the same volume.

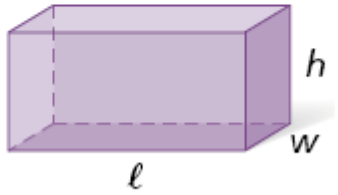
10-6 Volume of Prisms and Cylinders

Volume of a Prism

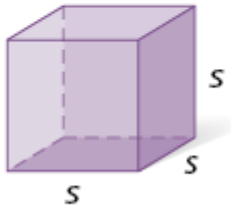
The volume of a prism with base area B and height h is $V = Bh$.



The volume of a right rectangular prism with length ℓ, width w, and height h is $V = \ell wh$.

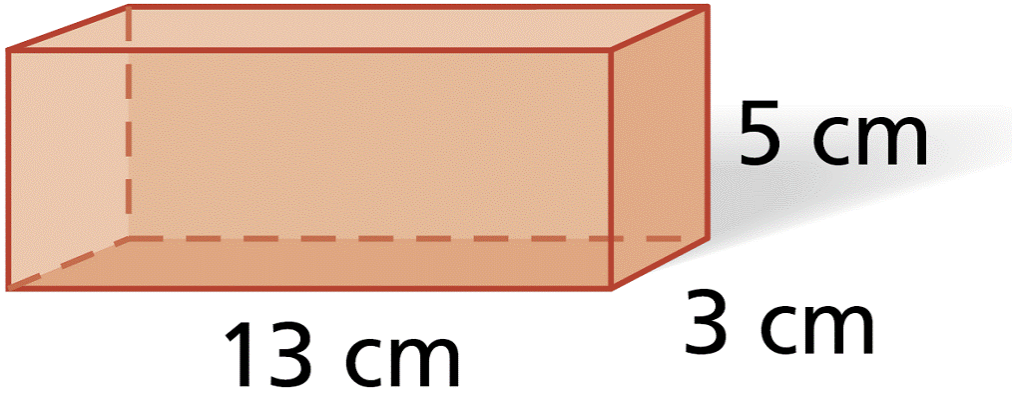


The volume of a cube with edge length s is $V = s^3$.



Example 1A: Finding Volumes of Prisms

Find the volume of the prism. Round to the nearest tenth, if necessary.



$V = \ell wh$ *Volume of a right rectangular prism*

$= (13)(3)(5)$ *Substitute 13 for ℓ, 3 for w, and 5 for h.*

$= 195 \text{ cm}^3$

Example 1B: Finding Volumes of Prisms

Find the volume of a cube with edge length 15 in. Round to the nearest tenth, if necessary.

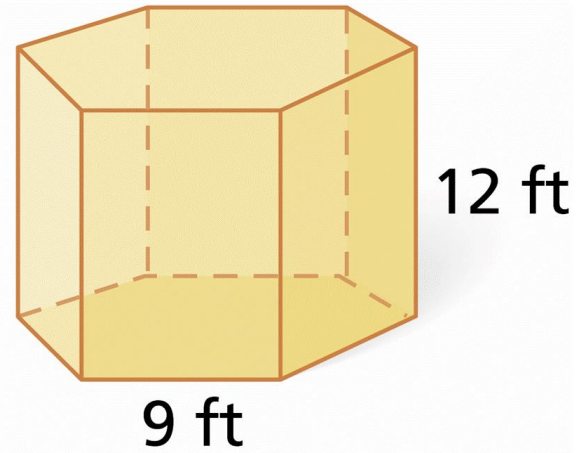
$V = s^3$ *Volume of a cube*

$= (15)^3$ *Substitute 15 for s.*

$= 3375 \text{ in}^3$

Example 1C: Finding Volumes of Prisms

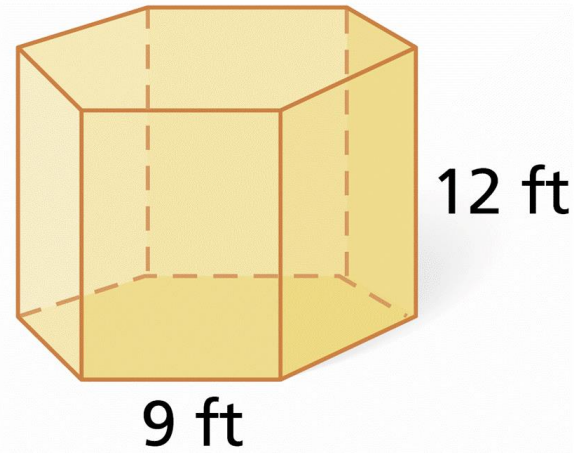
Find the volume of the right regular hexagonal prism. Round to the nearest tenth, if necessary.



Step 1 Find the apothem *a* of the base. First draw a right triangle on one base. The measure of the angle with its vertex at the center is $\frac{360°}{12} = 30°$.

Example 1C Continued

Find the volume of the right regular hexagonal prism. Round to the nearest tenth, if necessary.



So the sides are in ratio $1:\sqrt{3}:2$.

$$\frac{a}{4.5} = \frac{\sqrt{3}}{1}$$ *The leg of the triangle is half the side length, or 4.5 ft.*

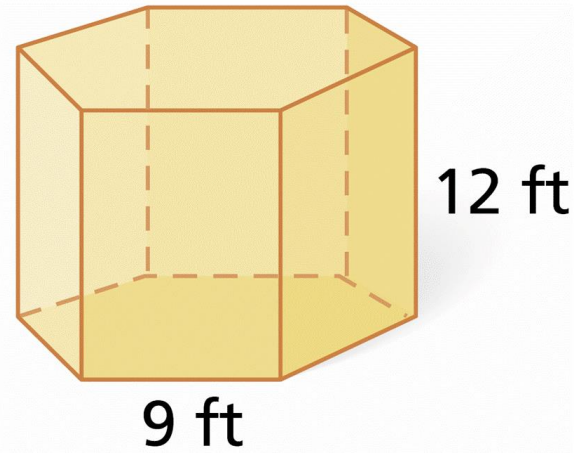
$$a = 4.5\sqrt{3}$$ *Solve for a.*

Step 2 Use the value of *a* to find the base area.

$$B = \frac{1}{2}aP = \frac{1}{2}\left(4.5\sqrt{3}\right)(54) = 121.5\sqrt{3}$$ *P = 6(9) = 54 ft*

Example 1C Continued

Find the volume of the right regular hexagonal prism. Round to the nearest tenth, if necessary.



Step 3 Use the base area to find the volume.

$$V = Bh = \left(121.5\sqrt{3}\right) \cdot 12 \approx 2525.3 \text{ ft}^2$$

Check It Out! Example 1

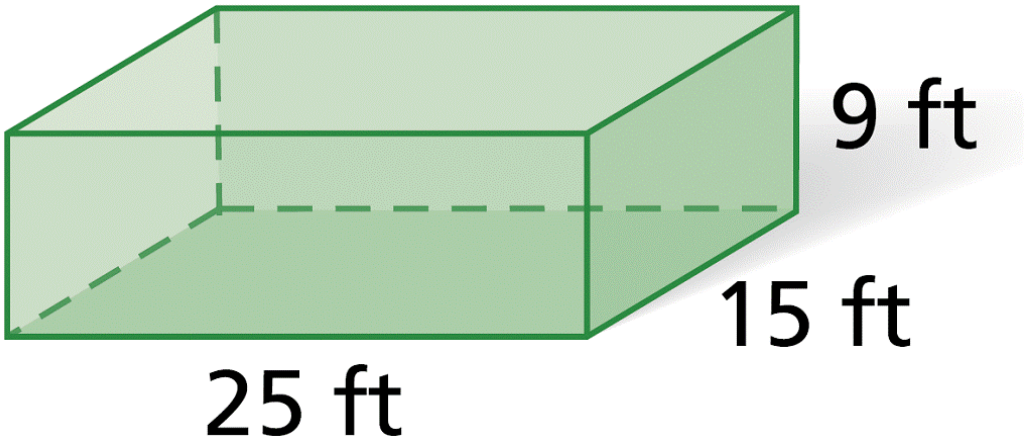
Find the volume of a triangular prism with a height of 9 yd whose base is a right triangle with legs 7 yd and 5 yd long.

$$V = \frac{1}{2}\ell wh$$ *Volume of a triangular prism*

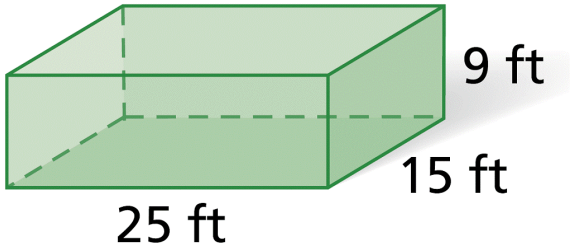
$$= \frac{1}{2}(5)(7)(9) = 157.5 \text{ yd}^3$$

Example 2: Recreation Application

A swimming pool is a rectangular prism. Estimate the volume of water in the pool in gallons when it is completely full (Hint: 1 gallon ≈ 0.134 ft^3). The density of water is about 8.33 pounds per gallon. Estimate the weight of the water in pounds.



Example 2 Continued



Step 1 Find the volume of the swimming pool in cubic feet.

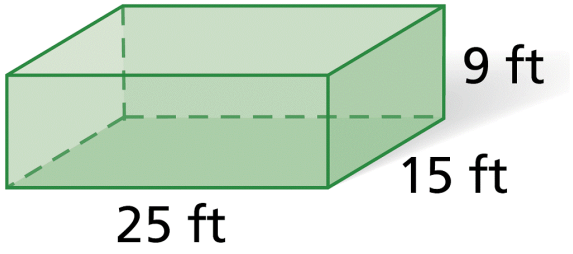
$$V = \ell wh = (25)(15)(19) = 3375 \text{ ft}^3$$

Step 2 Use the conversion factor $\frac{1 \text{ gallon}}{0.134 \text{ ft}^3}$ to estimate the volume in gallons.

$$3375 \text{ ft}^3 \cdot \frac{1 \text{ gallon}}{0.134 \text{ ft}^3} \approx 25{,}186.57 \text{ gallons}$$

$$\frac{1 \text{ gallon}}{0.134 \text{ ft}^3} = 1$$

Example 2 Continued



Step 3 Use the conversion factor $\frac{8.33 \text{ pounds}}{1 \text{ gallon}}$ to estimate the weight of the water.

$$25{,}186.57 \cdot \frac{8.33 \text{ pounds}}{1 \text{ gallon}}$$

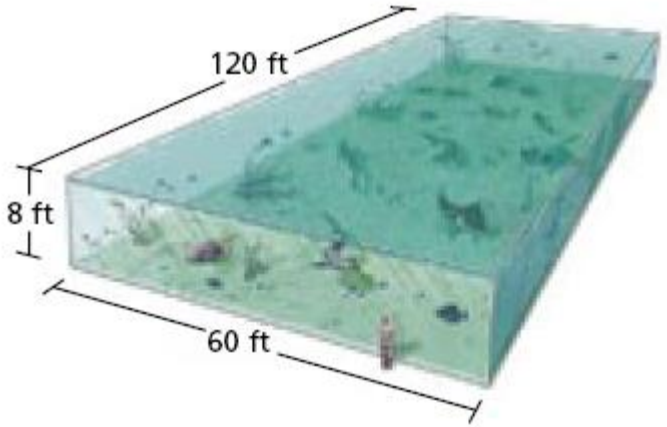
$$\frac{8.33 \text{ pounds}}{1 \text{ gallon}} = 1$$

$$\approx 209{,}804 \text{ pounds}$$

The swimming pool holds about 25,187 gallons. The water in the swimming pool weighs about 209,804 pounds.

Check It Out! Example 2

What if...? Estimate the volume in gallons and the weight of the water in the aquarium if the height were doubled.

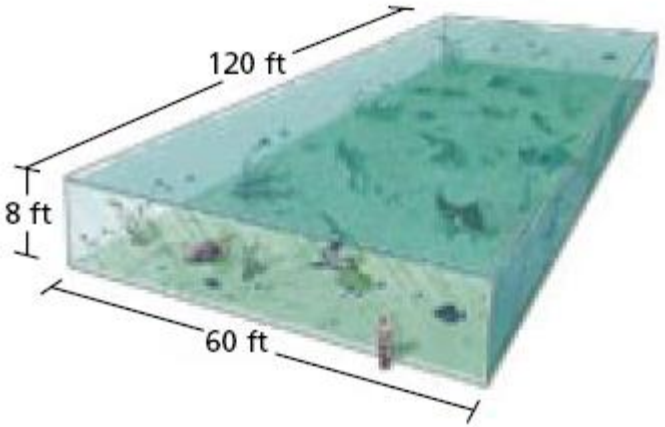


Step 1 Find the volume of the aquarium in cubic feet.

$V = \ell wh = (120)(60)(16) = 115{,}200 \text{ ft}^3$

Check It Out! Example 2 Continued

What if...? Estimate the volume in gallons and the weight of the water in the aquarium if the height were doubled.



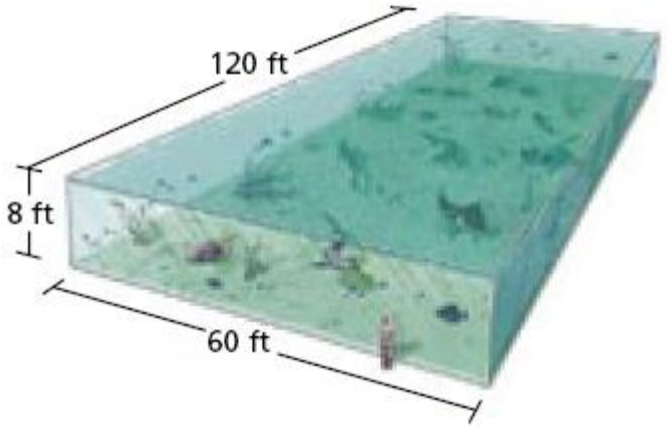
Step 2 Use the conversion factor $\frac{1 \text{ gallon}}{0.134 \text{ ft}^3}$ to estimate the volume in gallons.

$$115{,}200 \text{ ft}^3 \cdot \frac{1 \text{ gallon}}{0.134 \text{ ft}^3} \approx 859{,}701.49 \text{ gallons}$$

$$\frac{1 \text{ gallon}}{0.134 \text{ ft}^3} = 1$$

Check It Out! Example 2 Continued

What if...? Estimate the volume in gallons and the weight of the water in the aquarium if the height were doubled.



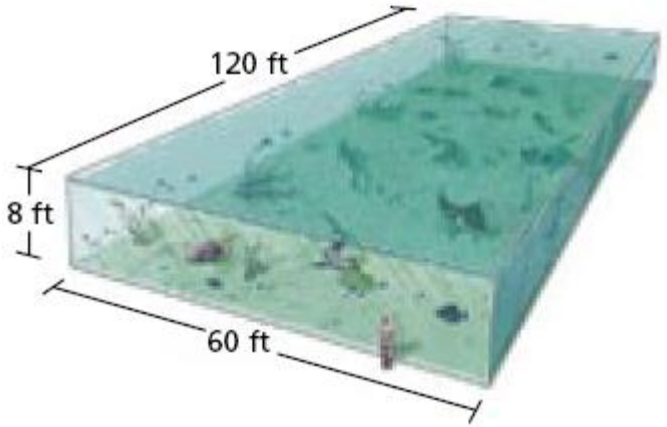
Step 3 Use the conversion factor $\frac{8.33 \text{ pounds}}{1 \text{ gallon}}$ to estimate the weight of the water.

$$\frac{8.33 \text{ pounds}}{1 \text{ gallon}} = 1$$

$$859{,}701.49 \cdot \frac{8.33 \text{ pounds}}{1 \text{ gallon}} \approx 7{,}161{,}313.41 \text{ pounds}$$

Check It Out! Example 2 Continued

What if...? Estimate the volume in gallons and the weight of the water in the aquarium if the height were doubled.



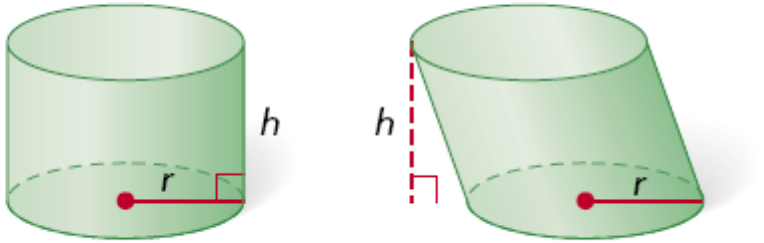
The swimming pool holds about 859,701 gallons. The water in the swimming pool weighs about 7,161,313 pounds.

10-6 Volume of Prisms and Cylinders

Cavalieri's principle also relates to cylinders. The two stacks have the same number of CDs, so they have the same volume.

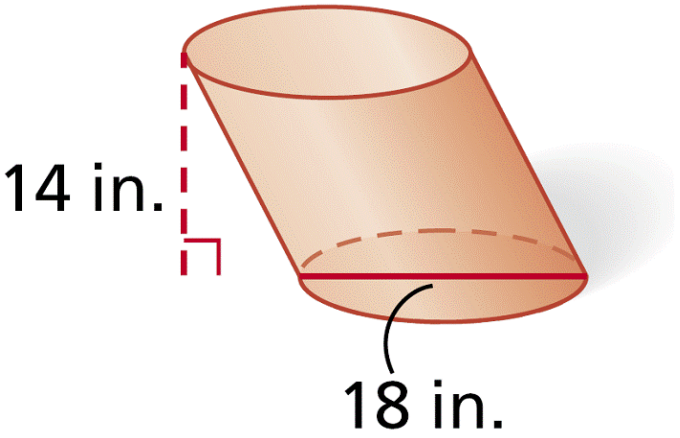
Volume of a Cylinder

The volume of a cylinder with base area B, radius r, and height h is $V = Bh$, or $V = \pi r^2 h$.



Example 3A: Finding Volumes of Cylinders

Find the volume of the cylinder. Give your answers in terms of π and rounded to the nearest tenth.



$V = \pi r^2 h$ *Volume of a cylinder*

$= \pi(9)^2(14)$ *Substitute* $\frac{18}{2} = 9$ *for r and 14 for h.*

$= 1134\pi \text{ in}^3 \approx 3562.6 \text{ in}^3$

Example 3B: Finding Volumes of Cylinders

Find the volume of a cylinder with base area 121π cm^2 and a height equal to twice the radius. Give your answer in terms of π and rounded to the nearest tenth.

Step 1 Use the base area to find the radius.

$\pi r^2 = 121\pi$ *Substitute 121π for the base area.*

$r = 11$ *Solve for r.*

Step 2 Use the radius to find the height. The height is equal to twice the radius.

$h = 2(r)$

$= 2(11) = 22$ cm

Example 3B Continued

Find the volume of a cylinder with base area π and a height equal to twice the radius. Give your answers in terms of π and rounded to the nearest tenth.

Step 3 Use the radius and height to find the volume.

$V = \pi r^2 h$ *Volume of a cylinder*

$= \pi(11)^2(22)$ *Substitute* $\frac{18}{2} = 9$ *for* r *and* 22 *for* h.

$= 2662\pi \text{ cm}^3 \approx 8362.9 \text{ cm}^3$

Check It Out! Example 3

Find the volume of a cylinder with a diameter of 16 in. and a height of 17 in. Give your answer both in terms of *π* and rounded to the nearest tenth.

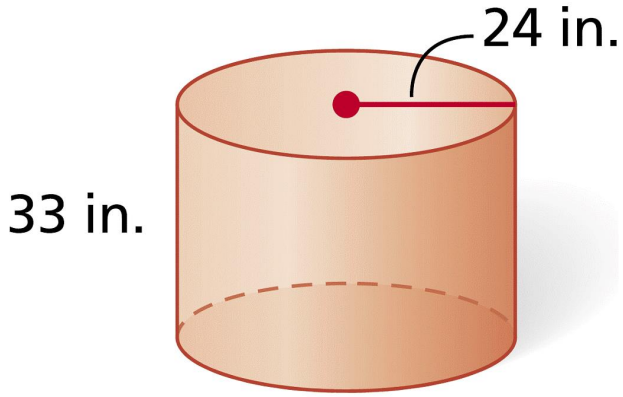
$V = \pi r^2 h$ *Volume of a cylinder*

$= \pi(8)^2(17)$ *Substitute 8 for r and 17 for h.*

$= 1088\pi \text{ in}^3 \approx 3418.1 \text{ in}^3$

Example 4: Exploring Effects of Changing Dimensions

The radius and height of the cylinder are multiplied by $\frac{2}{3}$. Describe the effect on the volume.



original dimensions:

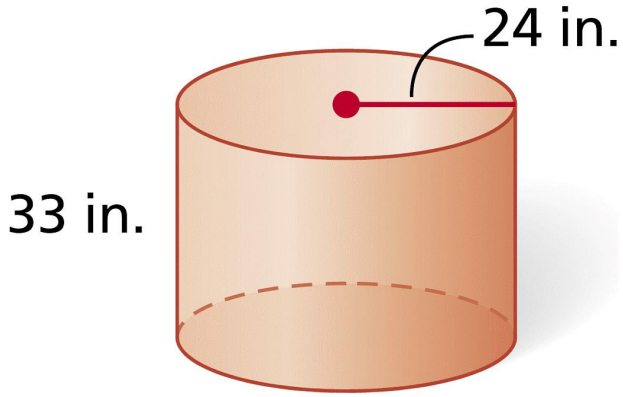
$$V = \pi r^2 h$$
$$= \pi(24)^2(33) = 19{,}008\pi \text{ in}^3$$

radius and height multiplied by $\frac{2}{3}$:

$$V = \pi r^2 h$$
$$= \pi(16)^2(22) = 5632\pi \text{ in}^3$$

Example 4 Continued

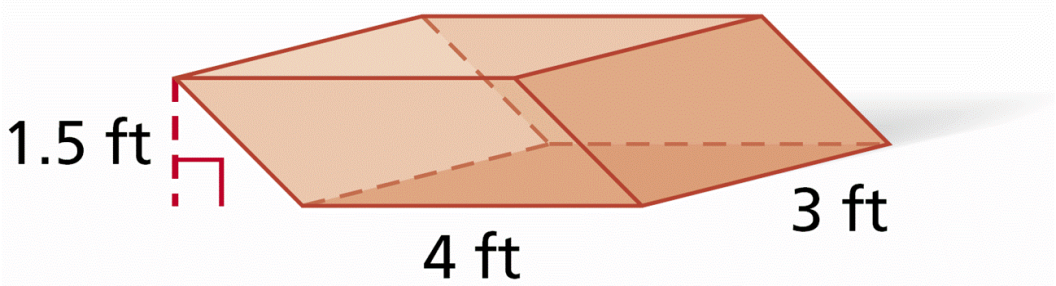
The radius and height of the cylinder are multiplied by $\frac{2}{3}$. Describe the effect on the volume.



Notice that $5632\pi = \frac{8}{27}19{,}008\pi$. If the radius and height are multiplied by $\frac{2}{3}$, the volume is multiplied by $\left(\frac{2}{3}\right)^3$, or $\frac{8}{27}$.

Check It Out! Example 4

The length, width, and height of the prism are doubled. Describe the effect on the volume.



original dimensions:

$V = \ell wh$
$= (1.5)(4)(3)$
$= 18$

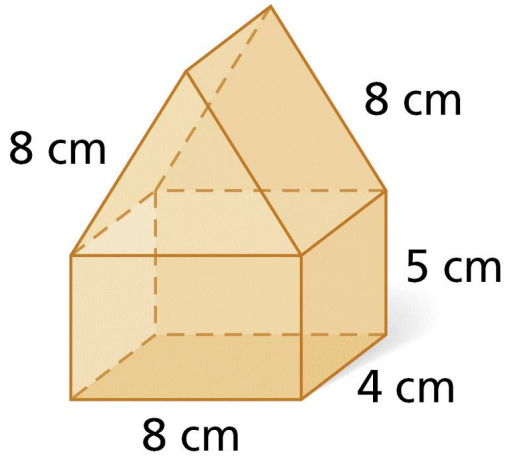
dimensions multiplied by 2:

$V = \ell wh$
$= (3)(8)(6)$
$= 144$

Doubling the dimensions increases the volume by 8 times.

Example 5: Finding Volumes of Composite Three-Dimensional Figures

Find the volume of the composite figure. Round to the nearest tenth.



The volume of the rectangular prism is:

$V = \ell wh = (8)(4)(5) = 160 \text{ cm}^3$

The base area of the regular triangular prism is:

$$B = \frac{1}{2}(8)\left(4\sqrt{3}\right) = 16\sqrt{3} \text{ cm}^2$$

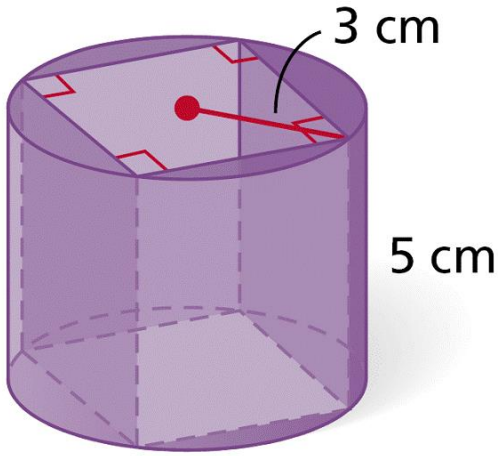
The volume of the regular triangular prism is:

$$V = Bh = \left(16\sqrt{3}\right)4 = 64\sqrt{3} \text{ cm}^3$$

The total volume of the figure is the sum of the volumes.

$$V = 160 + 64\sqrt{3} \approx 270.9 \text{ cm}^3$$

Check It Out! Example 5



Find the volume of the composite figure. Round to the nearest tenth.

Find the side length s of the base: $s = 3\sqrt{2}$

The volume of the square prism is:

$$V = s^2h = \left(3\sqrt{2}\right)^2(5) = 90$$

The volume of the cylinder is:

$$V = \pi r^2 h = \pi(3)^2(5) = 45\pi$$

The volume of the composite is the cylinder minus the rectangular prism.

$$V_{\text{cylinder}} - V_{\text{square prism}} = 45\pi - 90 \approx 51.4 \text{ cm}^3$$

Lesson Quiz: Part I

Find the volume of each figure. Round to the nearest tenth, if necessary.

1. a right rectangular prism with length 14 cm, width 11 cm, and height 18 cm $V = 2772\ \text{cm}^3$

2. a cube with edge length 22 ft $V = 10{,}648\ \text{ft}^3$

3. a regular hexagonal prism with edge length 10 ft and height 10 ft $V \approx 2598.1\ \text{ft}^3$

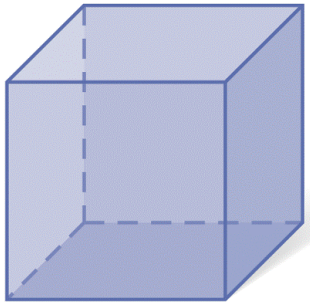
4. a cylinder with diameter 16 in. and height 7 in. $V \approx 1407.4\ \text{in}^3$

Lesson Quiz: Part II

5. a cylinder with base area 196π cm^2 and a height equal to the diameter $V \approx 17{,}241.1$ cm^3

6. The edge length of the cube is tripled. Describe the effect on the volume.

The volume is multiplied by 27.



7. Find the volume of the composite figure. Round to the nearest tenth.

9160.9 in^3