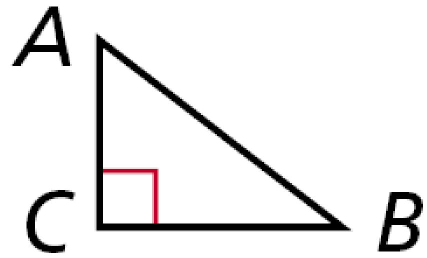
Warm Up

1. What are sides *AC* and *BC* called? Side *AB*?



legs; hypotenuse

2. Which side is in between $\angle A$ and $\angle C$?

$\overline{AC}$

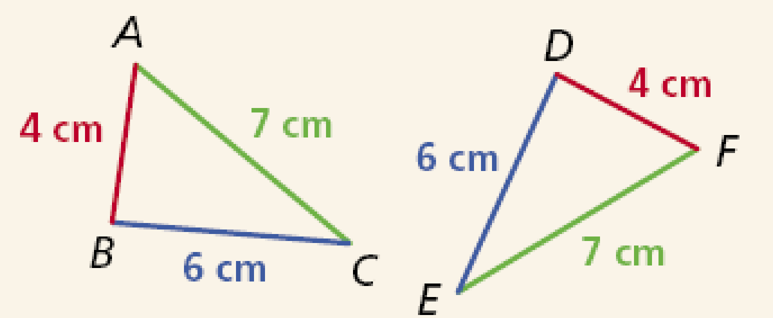
3. Given ΔDEF and ΔGHI, if $\angle D \cong \angle G$ and $\angle E \cong \angle H$, why is $\angle F \cong \angle I$?

Third $\angle$s Thm.

Objectives

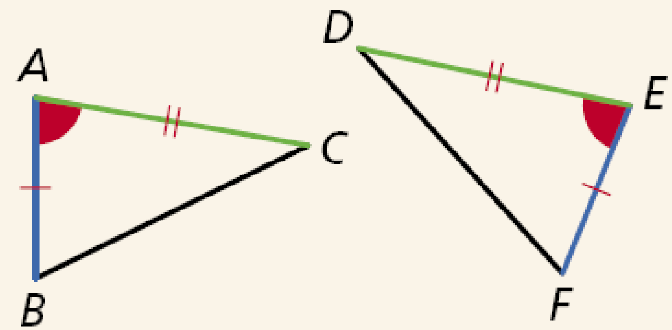
Prove triangles congruent by using ASA, AAS, and HL.

Side-Side-Side Triangle Congruence (SSS): If all pairs of corresponding sides between two triangles are congruent, then the triangles are congruent.

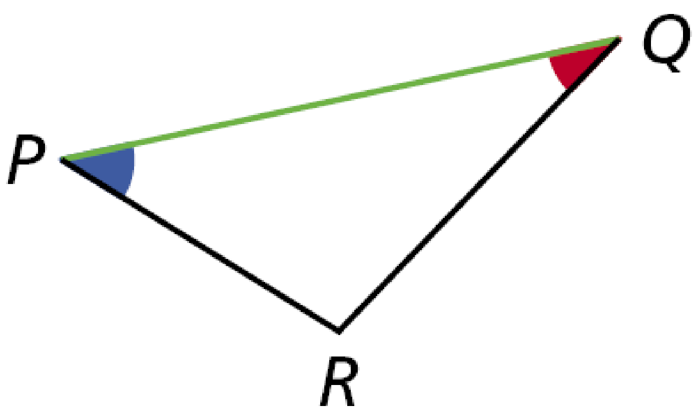
HYPOTHESIS	CONCLUSION
	$\triangle ABC \cong \triangle FDE$

by SSS

Postulate 4-4-2 Side-Angle-Side (SAS) Congruence

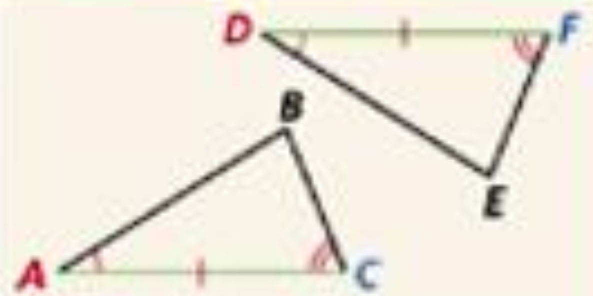
POSTULATE	HYPOTHESIS	CONCLUSION
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.		$\triangle ABC \cong \triangle EFD$

An **included side** is the common side of two consecutive angles in a polygon. The following postulate uses the idea of an included side.



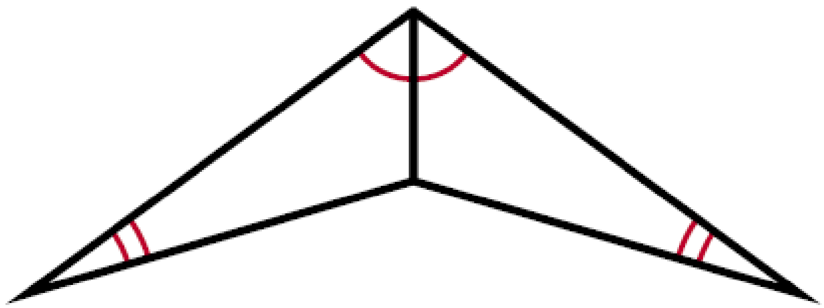
$\overline{PQ}$ is the included side of $\angle P$ and $\angle Q$.

Postulate 4-5-1 Angle-Side-Angle (ASA) Congruence

POSTULATE	HYPOTHESIS	CONCLUSION
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.		$\triangle ABC \cong \triangle DEF$

Example 2: Applying ASA Congruence

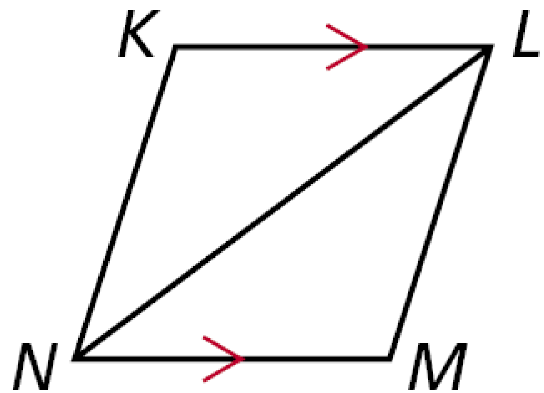
Determine if you can use ASA to prove the triangles congruent. Explain.



Two congruent angle pairs are give, but the included sides are not given as congruent. Therefore ASA cannot be used to prove the triangles congruent.

Check It Out! Example 2

Determine if you can use ASA to prove $\triangle NKL \cong \triangle LMN$. Explain.

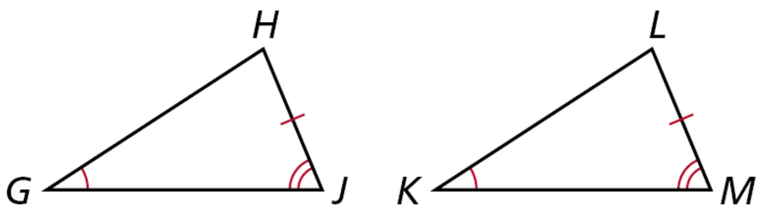


By the Alternate Interior Angles Theorem. $\angle KLN \cong \angle MNL$. $\overline{NL} \cong \overline{LN}$ by the Reflexive Property. No other congruence relationships can be determined, so ASA cannot be applied.

PROOF

Angle-Angle-Side Congruence

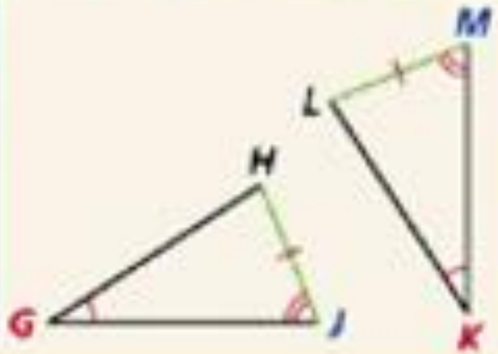
Given: $\angle G \cong \angle K$, $\angle J \cong \angle M$, $\overline{HJ} \cong \overline{LM}$
Prove: $\triangle GHJ \cong \triangle KLM$



Proof:

Statements	Reasons
1. $\angle G \cong \angle K$, $\angle J \cong \angle M$	1. Given
2. $\angle H \cong \angle L$	2. Third $\angle$s Thm.
3. $\overline{HJ} \cong \overline{LM}$	3. Given
4. $\triangle GHJ \cong \triangle KLM$	4. ASA *Steps 1, 3, and 2*

Theorem 4-5-2 Angle-Angle-Side (AAS) Congruence

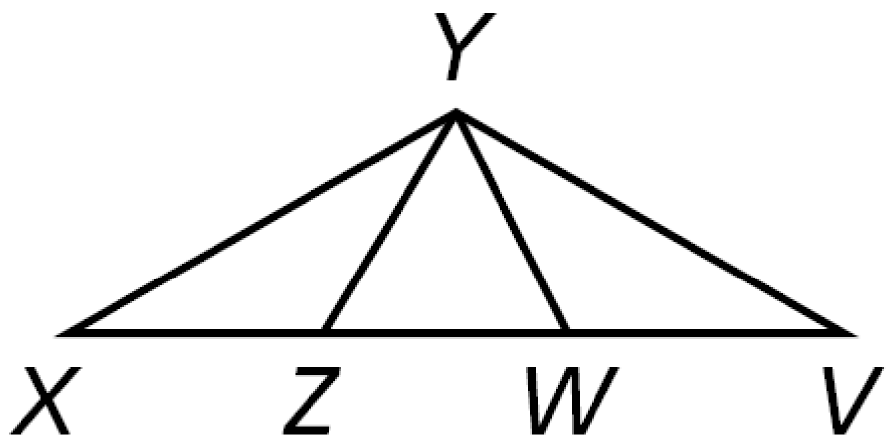
THEOREM	HYPOTHESIS	CONCLUSION
If two angles and a nonincluded side of one triangle are congruent to the corresponding angles and nonincluded side of another triangle, then the triangles are congruent.		$\triangle GHJ \cong \triangle KLM$

Example 3: Using AAS to Prove Triangles Congruent

Use AAS to prove the triangles congruent.

Given: $\angle X \cong \angle V$, $\angle YZW \cong \angle YWZ$, $\overline{XY} \cong \overline{VY}$

Prove: $\triangle XYZ \cong \triangle VYW$

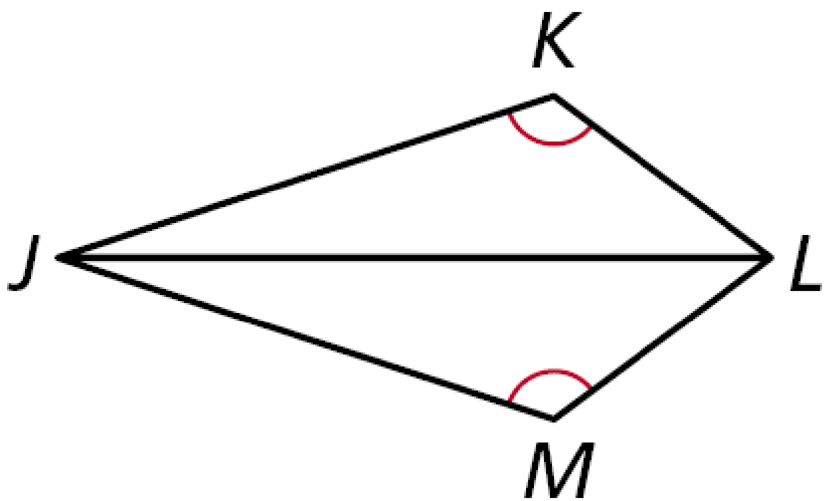


Check It Out! Example 3

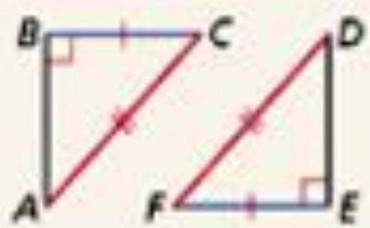
Use AAS to prove the triangles congruent.

Given: $\overline{JL}$ bisects $\angle KLM$, $\angle K \cong \angle M$

Prove: $\Delta JKL \cong \Delta JML$



Theorem 4-5-3 Hypotenuse-Leg (HL) Congruence

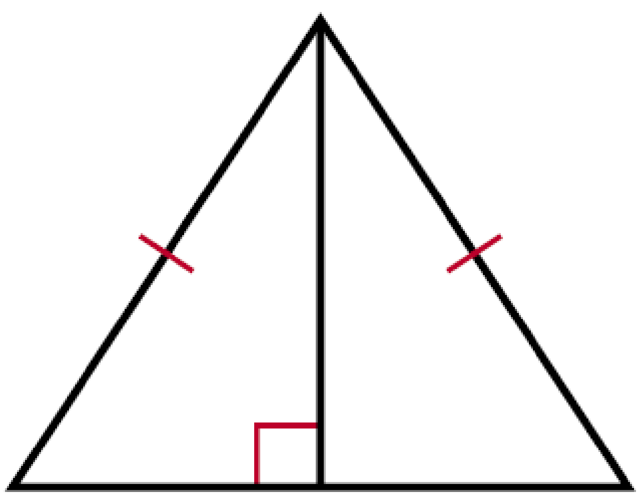
THEOREM	HYPOTHESIS	CONCLUSION
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.		$\triangle ABC \cong \triangle DEF$

Only works with Right Triangles!!!

Example 4A: Applying HL Congruence

Determine if you can use the HL Congruence Theorem to prove the triangles congruent. If not, tell what else you need to know.

A.

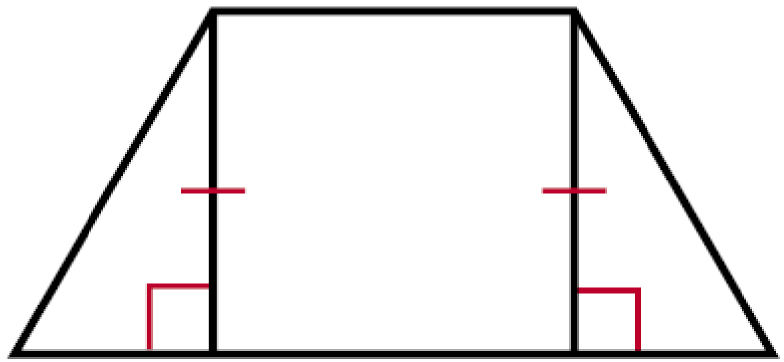


According to the diagram, the triangles are right triangles that share one leg.

It is given that the hypotenuses are congruent, therefore the triangles are congruent by HL.

Example 4B: Applying HL Congruence

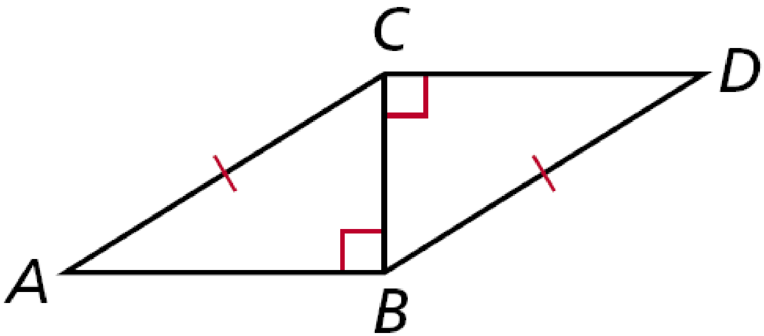
B.



This conclusion cannot be proved by HL. According to the diagram, the triangles are right triangles and one pair of legs is congruent. You do not know that one hypotenuse is congruent to the other.

Check It Out! Example 4

Determine if you can use the HL Congruence Theorem to prove $\triangle ABC \cong \triangle DCB$. If not, tell what else you need to know.

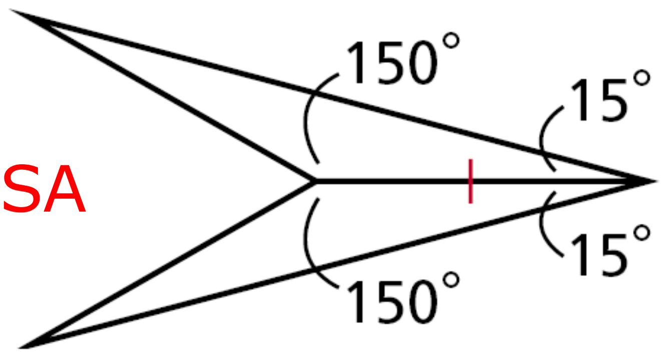


Yes; it is given that $\overline{AC} \cong \overline{DB}$. $\overline{BC} \cong \overline{CB}$ by the Reflexive Property of Congruence. Since $\angle ABC$ and $\angle DCB$ are right angles, $\triangle ABC$ and $\triangle DCB$ are right triangles. $\triangle ABC \cong DCB$ by *HL*.

Lesson Quiz: Part I

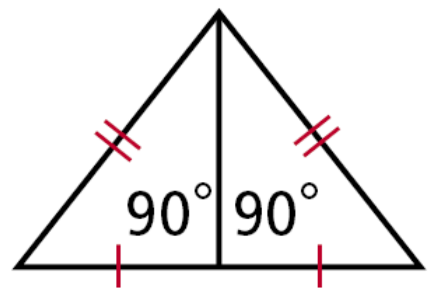
Identify the postulate or theorem that proves the triangles congruent.

1.



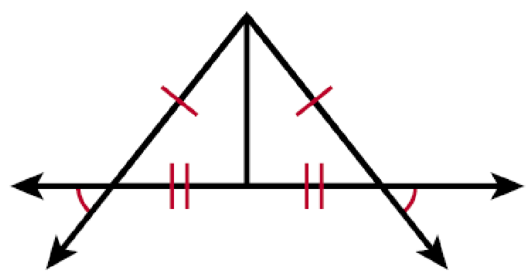
ASA

2.



HL

3.

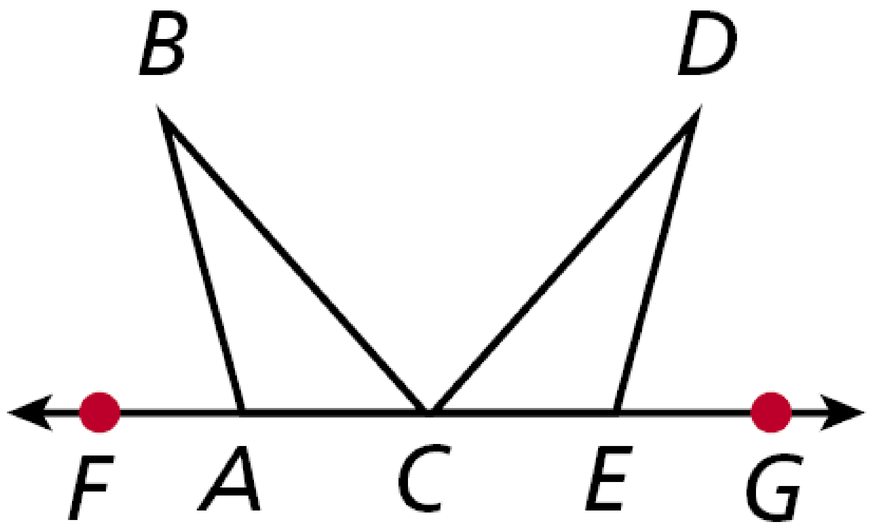


SAS or SSS

Lesson Quiz: Part II

4. Given: $\angle FAB \cong \angle GED$, $\angle ABC \cong \angle DCE$, $\overline{AC} \cong \overline{EC}$

Prove: $\triangle ABC \cong \triangle EDC$

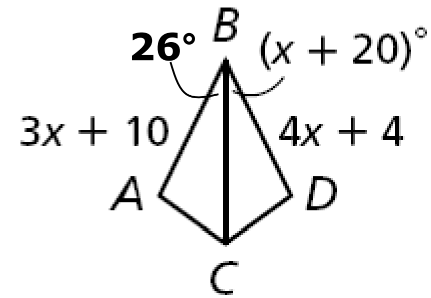


Lesson Quiz: Part II Continued

Statements	Reasons
1. $\angle FAB \cong \angle GED$	**1.** Given
2. $\angle BAC$ is a supp. of $\angle FAB$; $\angle DEC$ is a supp. of $\angle GED$.	**2.** Def. of supp. $\angle$s
3. $\angle BAC \cong \angle DEC$	**3.** $\cong$ Supp. Thm.
4. $\angle ACB \cong \angle DCE$; $\overline{AC} \cong \overline{EC}$	**4.** Given
5. $\Delta ABC \cong \Delta EDC$	**5.** ASA Steps 3,4

Lesson Quiz: Part I

1. Show that $\Delta ABC \cong \Delta DBC$, when $x = 6$.



$\angle ABC \cong \angle DBC$

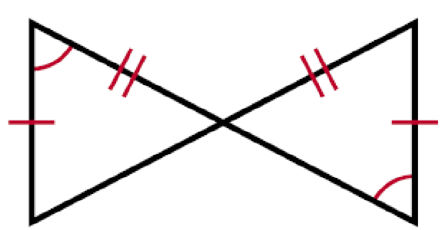
$\overline{BC} \cong \overline{BC}$

$\overline{AB} \cong \overline{DB}$

So $\Delta ABC \cong \Delta DBC$ by SAS

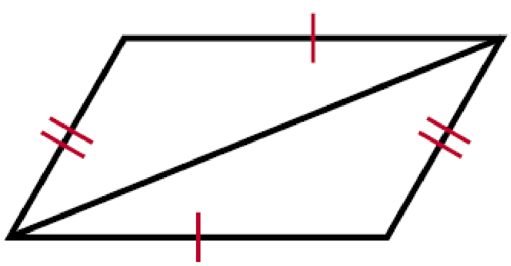
Which postulate, if any, can be used to prove the triangles congruent?

2.



none

3.

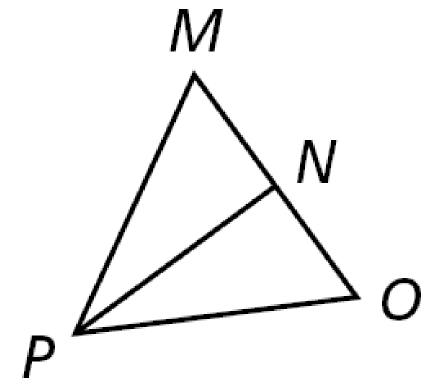


SSS

Lesson Quiz: Part II

4. Given: $\overline{PN}$ bisects $\overline{MO}$, $PN \perp MO$

Prove: $\Delta MNP \cong \Delta ONP$



Statements	Reasons
1. $\overline{PN}$ bisects $\overline{MO}$	**1.** Given
2. $\overline{MN} \cong \overline{ON}$	**2.** Def. of bisect
3. $\overline{PN} \cong \overline{PN}$	**3.** Reflex. Prop. of $\cong$
4. $\overline{PN} \perp \overline{MO}$	**4.** Given
5. $\angle PNM$ and $\angle PNO$ are rt. $\angle$s	**5.** Def. of $\perp$
6. $\angle PNM \cong \angle PNO$	**6.** Rt. $\angle \cong$ Thm.
7. $\Delta MNP \cong \Delta ONP$	**7.** SAS Steps 2, 6, 3