Warm Up

1. Name the angle formed by $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

Possible answer: $\angle A$

2. Name the three sides of ΔABC.

$\overline{AB}, \overline{AC}, \overline{BC}$

3. $\Delta QRS \cong \Delta LMN$. Name all pairs of congruent corresponding parts.

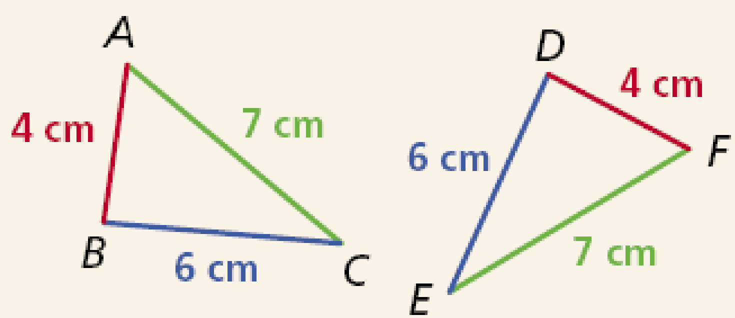
$\overline{QR} \cong \overline{LM}$, $\overline{RS} \cong \overline{MN}$, $\overline{QS} \cong \overline{LN}$, $\angle Q \cong \angle L$, $\angle R \cong \angle M$, $\angle S \cong \angle N$

Objectives

Prove triangles congruent by using SSS and SAS.

Side-Side-Side Triangle Congruence (SSS): If all pairs of corresponding sides between two triangles are congruent, then the triangles are congruent.

HYPOTHESIS



CONCLUSION

$\triangle ABC \cong \triangle FDE$

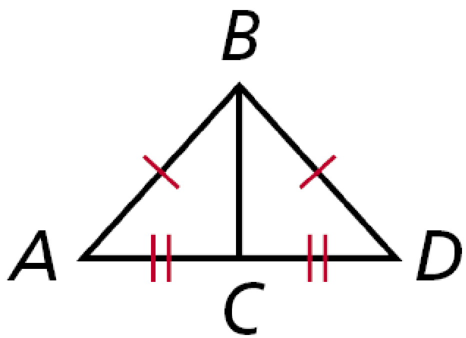
by SSS

Remember!

Adjacent triangles share a side, so you can apply the Reflexive Property to get a pair of congruent parts.

Example 1: Using SSS to Prove Triangle Congruence

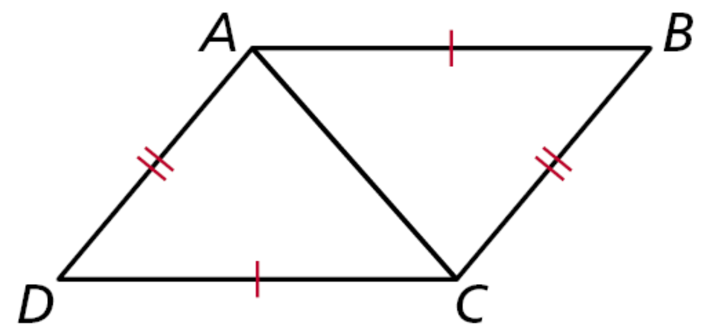
Prove $\Delta ABC \cong \Delta DBC$ using given info from the picture.



It is given that $\overline{AC} \cong \overline{DC}$ and that $\overline{AB} \cong \overline{DB}$. By the Reflexive Property of Congruence, $\overline{BC} \cong \overline{BC}$. Therefore $\Delta ABC \cong \Delta DBC$ by SSS.

Check It Out! Example 1

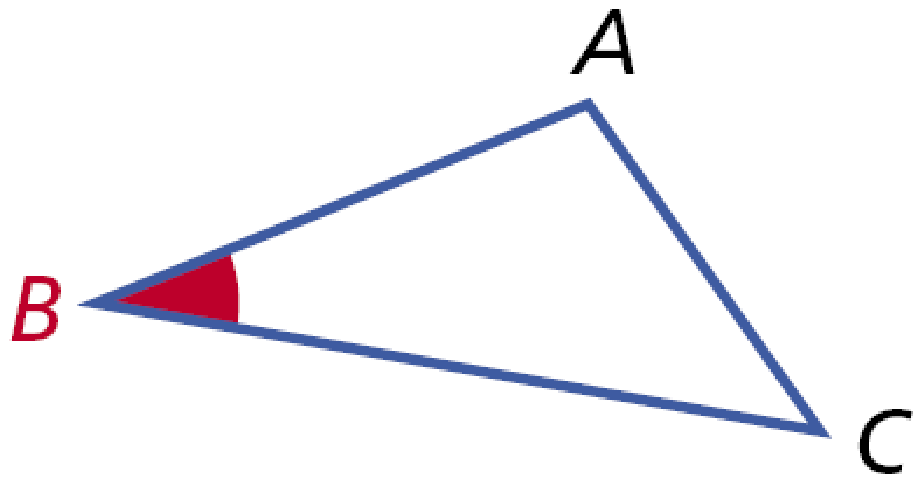
Prove $\Delta ABC \cong \Delta CDA$.



It is given that $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$.

By the Reflexive Property of Congruence, $\overline{AC} \cong \overline{CA}$.

So $\Delta ABC \cong \Delta CDA$ by SSS.

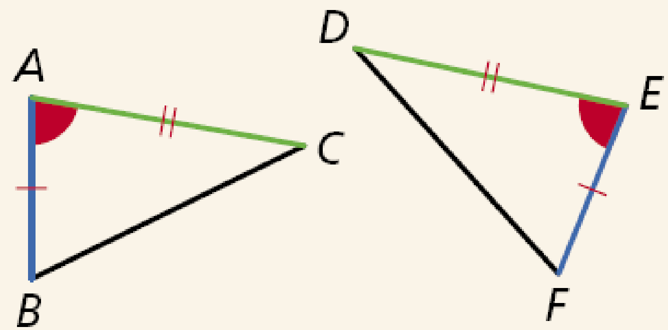


An **included angle** is an angle formed by two adjacent sides of a polygon.

$\angle B$ is the included angle between sides $\overline{AB}$ and $\overline{BC}$.

It can also be shown that only two pairs of congruent corresponding sides are needed to prove the congruence of two triangles if the included angles are also congruent.

Postulate 4-4-2 Side-Angle-Side (SAS) Congruence

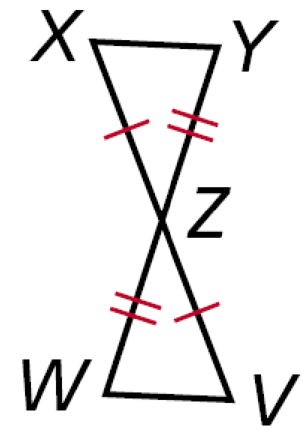
POSTULATE	HYPOTHESIS	CONCLUSION
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.		$\triangle ABC \cong \triangle EFD$

Caution

The letters SAS are written in that order because the congruent angles must be between pairs of congruent corresponding sides.

Example 2: Engineering Application

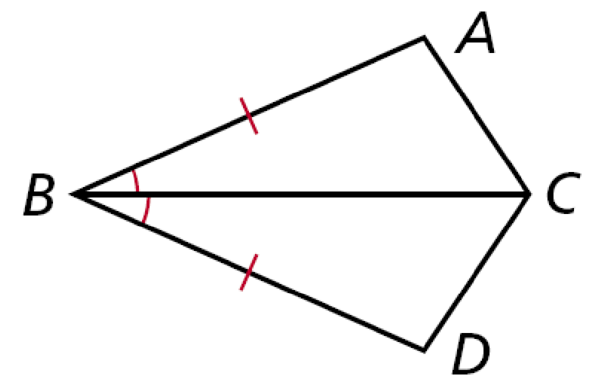
Prove $\triangle XYZ \cong \triangle VWZ$.



It is given that $\overline{XZ} \cong \overline{VZ}$ and that $\overline{YZ} \cong \overline{WZ}$. By the Vertical ∠s Theorem. $\angle XZY \cong \angle VZW$. Therefore $\triangle XYZ \cong \triangle VWZ$ by SAS.

Check It Out! Example 2

Prove $\triangle ABC \cong \triangle DBC$.

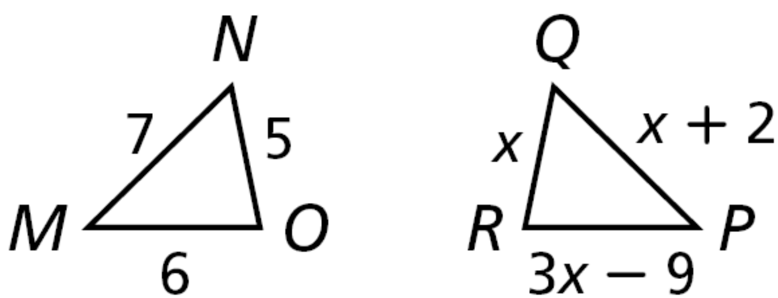


It is given that $\overline{BA} \cong \overline{BD}$ and $\angle ABC \cong \angle DBC$.
By the Reflexive Property of $\cong$, $\overline{BC} \cong \overline{BC}$.
So $\triangle ABC \cong \triangle DBC$ by SAS.

Example 3A: Verifying Triangle Congruence

Show that the triangles are congruent for the given value of the variable.

$\Delta MNO \cong \Delta PQR$, when $x = 5$.



$PQ = x + 2$

$= 5 + 2 = 7$

$QR = x = 5$

$PR = 3x - 9$

$= 3(5) - 9 = 6$

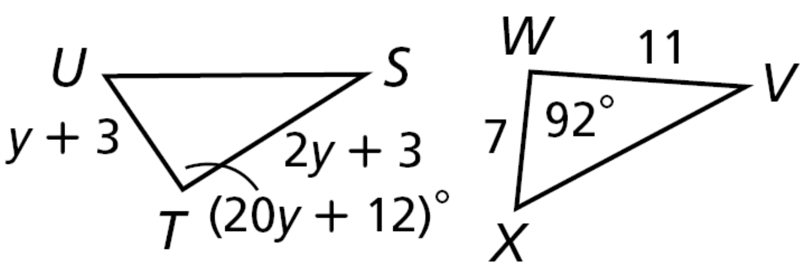
$\overline{PQ} \cong \overline{MN}$, $\overline{QR} \cong \overline{NO}$, $\overline{PR} \cong \overline{MO}$

$\Delta MNO \cong \Delta PQR$ by SSS.

Example 3B: Verifying Triangle Congruence

Show that the triangles are congruent for the given value of the variable.

$\Delta STU \cong \Delta VWX$, when $y = 4$.



$ST = 2y + 3$

$= 2(4) + 3 = 11$

$TU = y + 3$

$= 4 + 3 = 7$

$m\angle T = 20y + 12$

$= 20(4)+12 = 92°$

$\overline{ST} \cong \overline{VW}$, $\overline{TU} \cong \overline{WX}$, and $\angle T \cong \angle W$.

$\Delta STU \cong \Delta VWX$ by SAS.

Check It Out! Example 3

Show that $\Delta ADB \cong \Delta CDB$, $t = 4$.

$DA = 3t + 1$

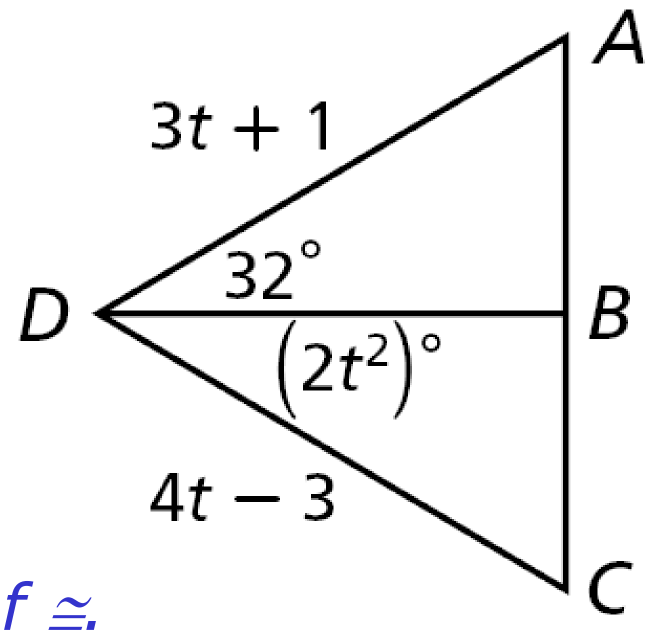
$= 3(4) + 1 = 13$

$DC = 4t - 3$

$= 4(4) - 3 = 13$

$m\angle D = 2t^2$

$= 2(16) = 32°$



$\angle ADB \cong \angle CDB$ *Def. of $\cong$.*

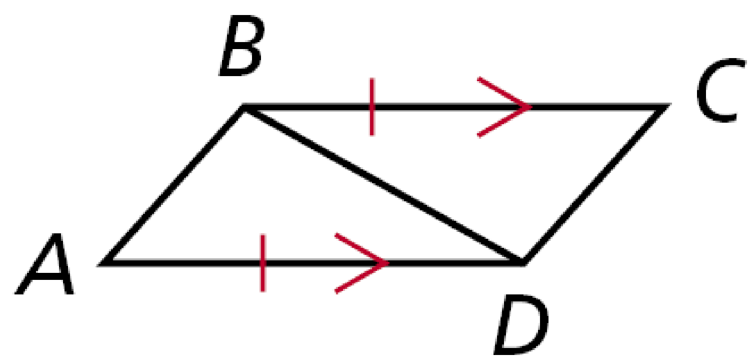
$\overline{DB} \cong \overline{DB}$ *Reflexive Prop. of $\cong$.*

$\Delta ADB \cong \Delta CDB$ by SAS.

Example 4: Proving Triangles Congruent

Given: $BC \parallel AD$, $\overline{BC} \cong \overline{AD}$

Prove: $\Delta ABD \cong \Delta CDB$

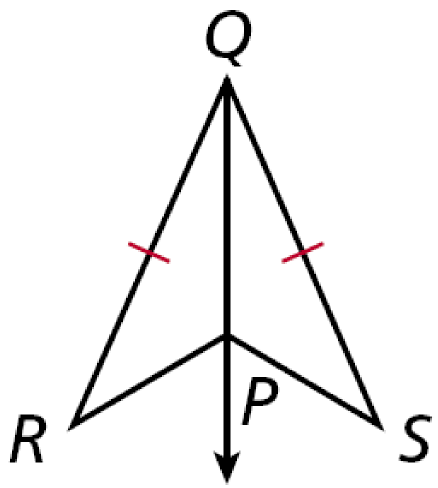


Statements	Reasons
1. $\overline{BC} \parallel \overline{AD}$	**1.** Given
2. $\angle CBD \cong \angle ABD$	**2.** Alt. Int. $\angle$s Thm.
3. $\overline{BC} \cong \overline{AD}$	**3.** Given
4. $\overline{BD} \cong \overline{BD}$	**4.** Reflex. Prop. of $\cong$
5. $\Delta ABD \cong \Delta\ CDB$	**5.** SAS Steps 3, 2, 4

Check It Out! Example 4

Given: $\overrightarrow{QP}$ bisects $\angle RQS$. $\overline{QR} \cong \overline{QS}$

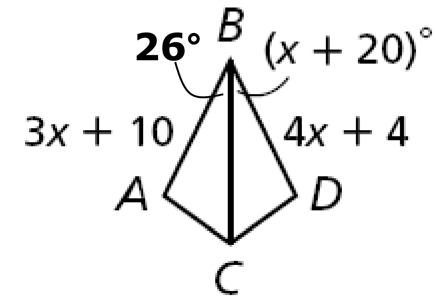
Prove: $\Delta RQP \cong \Delta SQP$



Statements	Reasons
1. $\overline{QR} \cong \overline{QS}$	**1.** Given
2. $\overrightarrow{QP}$ bisects $\angle RQS$	**2.** Given
3. $\angle RQP \cong \angle SQP$	**3.** Def. of bisector
4. $\overline{QP} \cong \overline{QP}$	**4.** Reflex. Prop. of $\cong$
5. $\Delta RQP \cong \Delta SQP$	**5.** SAS Steps 1, 3, 4

Lesson Quiz: Part I

1. Show that $\Delta ABC \cong \Delta DBC$, when $x = 6$.



$\angle ABC \cong \angle DBC$

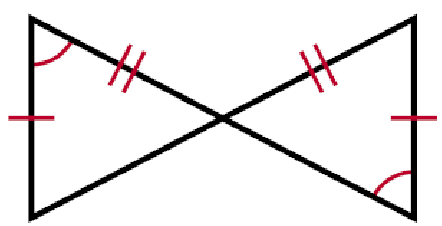
$\overline{BC} \cong \overline{BC}$

$\overline{AB} \cong \overline{DB}$

So $\Delta ABC \cong \Delta DBC$ by SAS

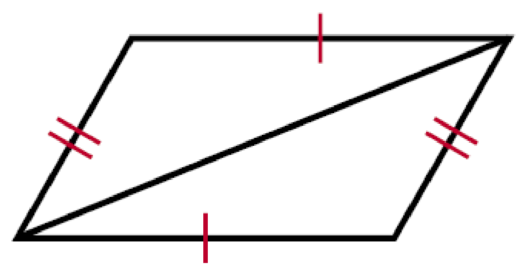
Which postulate, if any, can be used to prove the triangles congruent?

2.



none

3.

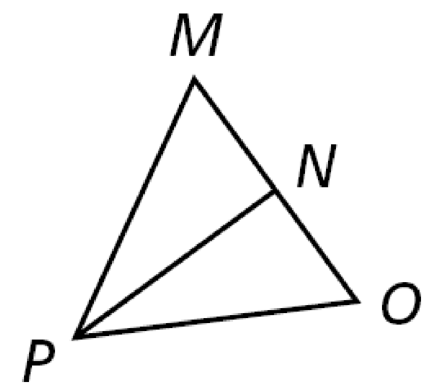


SSS

Lesson Quiz: Part II

4. Given: $\overline{PN}$ bisects $\overline{MO}$, $PN \perp MO$

Prove: $\Delta MNP \cong \Delta ONP$



Statements	Reasons
1. $\overline{PN}$ bisects $\overline{MO}$	**1.** Given
2. $\overline{MN} \cong \overline{ON}$	**2.** Def. of bisect
3. $\overline{PN} \cong \overline{PN}$	**3.** Reflex. Prop. of $\cong$
4. $\overline{PN} \perp \overline{MO}$	**4.** Given
5. $\angle PNM$ and $\angle PNO$ are rt. $\angle$s	**5.** Def. of $\perp$
6. $\angle PNM \cong \angle PNO$	**6.** Rt. $\angle \cong$ Thm.
7. $\Delta MNP \cong \Delta ONP$	**7.** SAS Steps 2, 6, 3

- **HOMEWORK**
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