BELLWORK
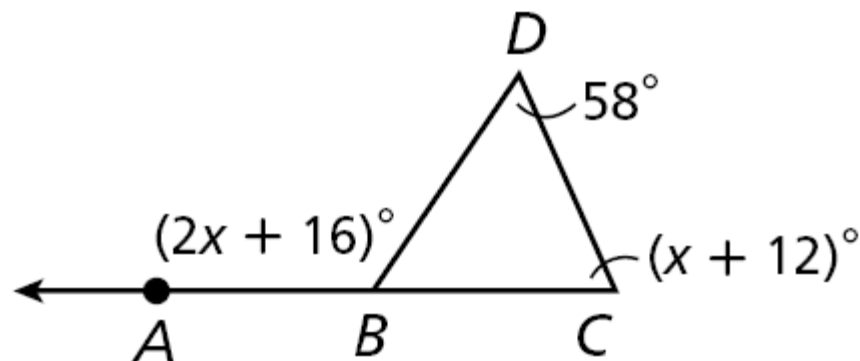
1. The measure of one of the acute angles in a right triangle is $56\frac{2}{3}°$. What is the measure of the other acute angle? $33\frac{1}{3}°$

2. Find m$\angle ABD$.

124°

Objectives

Use properties of congruent triangles.

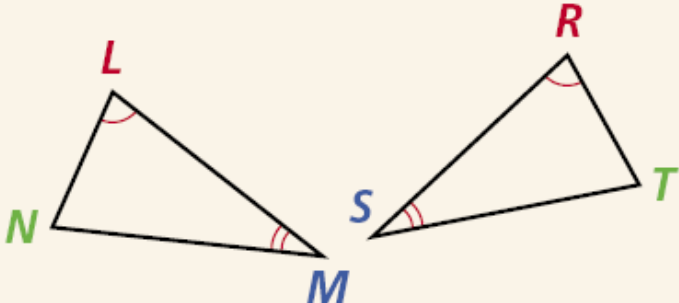Prove triangles congruent by using the definition of congruence.

Vocabulary

corresponding angles

corresponding sides

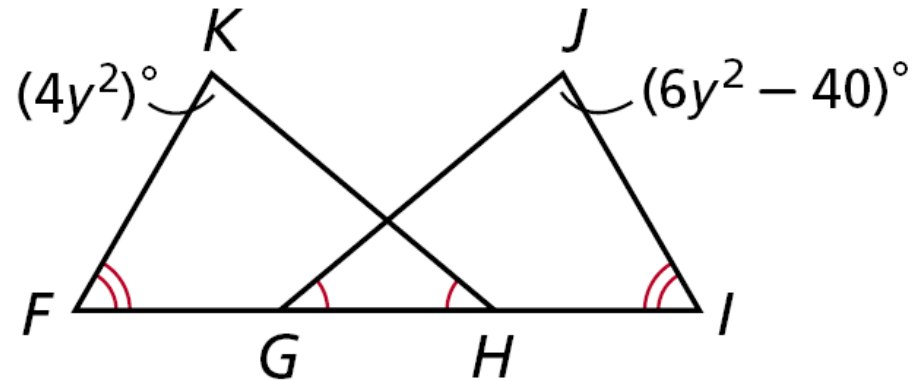congruent polygons

Third Angles Theorem:

THEOREM	HYPOTHESIS	CONCLUSION
If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.		$\angle N \cong \angle T$

Example 4: Applying the Third Angles Theorem

Find m∠K and m∠J.

$\angle K \cong \angle J$ *Third ∠s Thm.*

$m\angle K = m\angle J$ *Def. of ≅ ∠s.*

$4y^2 = 6y^2 - 40$ *Substitute $4y^2$ for m∠K and $6y^2 - 40$ for m∠J.*

$-2y^2 = -40$ *Subtract $6y^2$ from both sides.*

$y^2 = 20$ *Divide both sides by -2.*

So m∠K = $4y^2$ = 4(20) = 80°.

Since m∠J = m∠K, m∠J = 80°.

Check It Out! Example 4

Find m∠P and m∠T.

P

$(2x^2)°$

R

T

M

$(4x^2 - 32)°$

S

N

∠P ≅ ∠T   *Third ∠s Thm.*

m∠P = m∠T   *Def. of ≅ ∠s.*
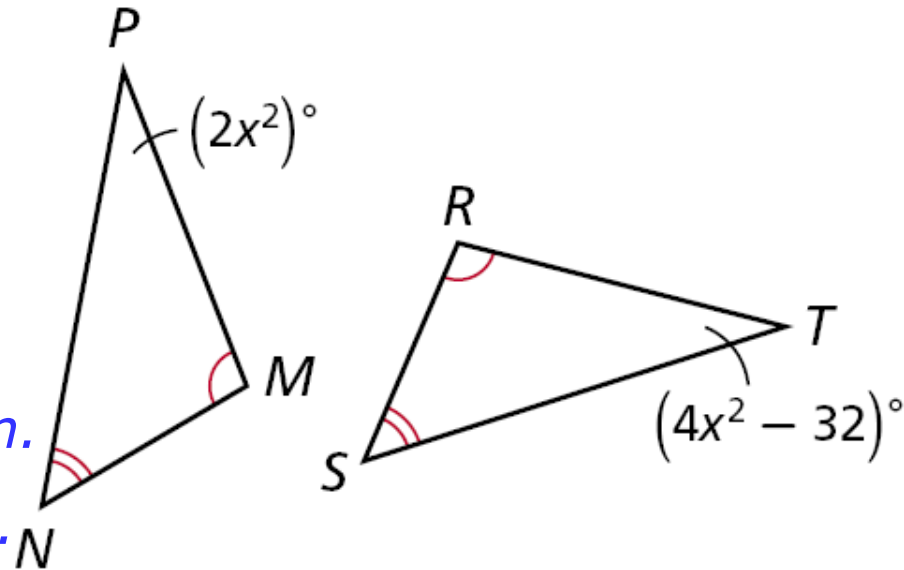
$2x^2 = 4x^2 - 32$ *Substitute $2x^2$ for m∠P and $4x^2 - 32$ for m∠T.*

$-2x^2 = -32$ *Subtract $4x^2$ from both sides.*

$x^2 = 16$ *Divide both sides by -2.*

So m∠P = $2x^2$ = 2(16) = 32°.

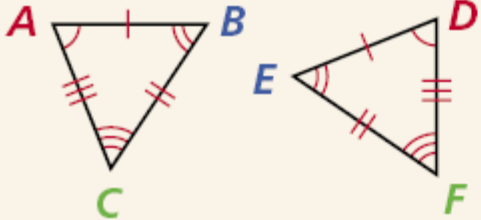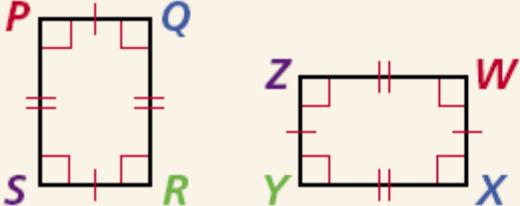Since m∠P = m∠T, m∠T = 32°.

Geometric figures are congruent if they are the same size and shape.

Corresponding angles and **corresponding sides:** In the same position on a different figure of the same shape.

Congruent polygons - corresponding sides and angles are congruent.

Properties of Congruent Polygons

DIAGRAM	CORRESPONDING ANGLES	CORRESPONDING SIDES
 △ABC ≅ △DEF	$\angle A \cong \angle D$ $\angle B \cong \angle E$ $\angle C \cong \angle F$	$\overline{AB} \cong \overline{DE}$ $\overline{BC} \cong \overline{EF}$ $\overline{AC} \cong \overline{DF}$
 polygon PQRS ≅ polygon WXYZ	$\angle P \cong \angle W$ $\angle Q \cong \angle X$ $\angle R \cong \angle Y$ $\angle S \cong \angle Z$	$\overline{PQ} \cong \overline{WX}$ $\overline{QR} \cong \overline{XY}$ $\overline{RS} \cong \overline{YZ}$ $\overline{PS} \cong \overline{WZ}$

Helpful Hint

Two vertices that are the endpoints of a side are called consecutive vertices.

For example, *P* and *Q* are consecutive vertices.

To name a polygon, write the vertices in consecutive order. For example, you can name polygon *PQRS* as *QRSP* or *SRQP*, but **not** as *PRQS*.

In a congruence statement, pay attention to **the order of the vertices;** it indicates the corresponding parts.

Helpful Hint

When you write a statement such as $\triangle ABC \cong \triangle DEF$, you are also stating which parts are congruent.

Example 1: Naming Congruent Corresponding Parts

Given: ΔPQR ≅ ΔSTW

Identify all pairs of corresponding congruent parts.

Angles: $\angle P \cong \angle S$, $\angle Q \cong \angle T$, $\angle R \cong \angle W$

Sides: $\overline{PQ} \cong \overline{ST}$, $\overline{QR} \cong \overline{TW}$, $\overline{PR} \cong \overline{SW}$

If polygon *LMNP* ≅ polygon *EFGH*, identify all pairs of corresponding congruent parts.

Angles: $\angle L \cong \angle E$, $\angle M \cong \angle F$, $\angle N \cong \angle G$, $\angle P \cong \angle H$

Sides: $\overline{LM} \cong \overline{EF}$, $\overline{MN} \cong \overline{FG}$, $\overline{NP} \cong \overline{GH}$, $\overline{LP} \cong \overline{EH}$

Example 2A: Using Corresponding Parts of Congruent Triangles

Given: $\triangle ABC \cong \triangle DBC$.

Find the value of x.
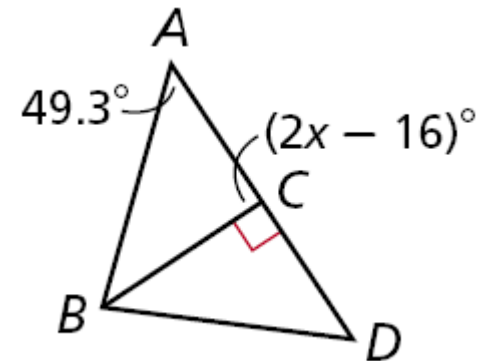
$\angle BCA$ and $\angle BCD$ are rt. $\angle$s.	*Def. of $\perp$ lines.*
$\angle BCA \cong \angle BCD$	*Rt. $\angle \cong$ Thm.*
m$\angle BCA$ = m$\angle BCD$	*Def. of $\cong \angle$s*
$(2x - 16)° = 90°$	*Substitute values for m$\angle BCA$ and m$\angle BCD$.*
$2x = 106$	*Add 16 to both sides.*
$x = 53$	*Divide both sides by 2.*

Example 2B: Using Corresponding Parts of Congruent Triangles

Given: $\triangle ABC \cong \triangle DBC$.

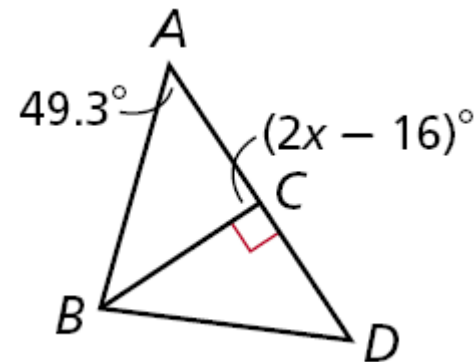Find m$\angle DBC$.

m$\angle ABC$ + m$\angle BCA$ + m$\angle A$ = 180°   *Δ Sum Thm.*

m$\angle ABC$ + 90 + 49.3 = 180   *Substitute values for m$\angle BCA$ and m$\angle A$.*

m$\angle ABC$ + 139.3 = 180   *Simplify.*

m$\angle ABC$ = 40.7   *Subtract 139.3 from both sides.*

$\angle DBC \cong \angle ABC$ *Corr. $\angle$s of $\cong \Delta$s are $\cong$.*

m$\angle DBC$ = m$\angle ABC$ *Def. of $\cong \angle$s.*

m$\angle DBC$ = 40.7°   *Trans. Prop. of =*

Check It Out! Example 2a

Given: $\Delta ABC \cong \Delta DEF$

Find the value of x.

$\overline{AB} \cong \overline{DE}$ — *Corr. sides of $\cong$ Δs are $\cong$.*
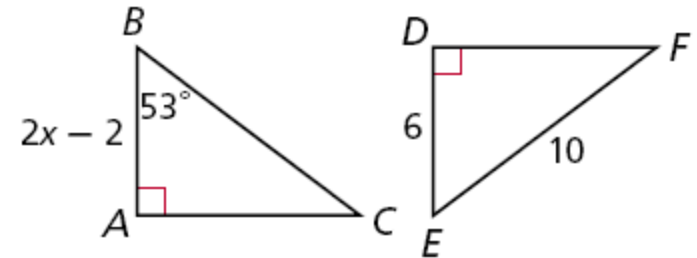
$AB = DE$ — *Def. of $\cong$ parts.*

$2x - 2 = 6$ — *Substitute values for AB and DE.*

$2x = 8$ — *Add 2 to both sides.*

$x = 4$ — *Divide both sides by 2.*

Check It Out! Example 2b

Given: $\Delta ABC \cong \Delta DEF$

Find m$\angle F$.
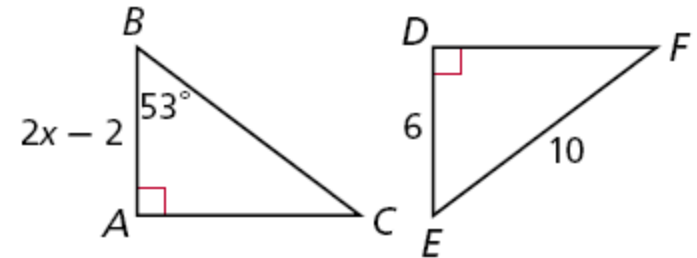
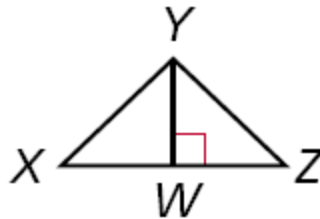|	
m$\angle EFD$ + m$\angle DEF$ + m$\angle FDE$ = 180°	*Δ Sum Thm.*
$\angle ABC \cong \angle DEF$	*Corr. $\angle$s of $\cong$ Δ are $\cong$.*
m$\angle ABC$ = m$\angle DEF$	*Def. of $\cong$ $\angle$s.*
m$\angle DEF$ = 53°	*Transitive Prop. of =.*
m$\angle EFD$ + 53 + 90 = 180	*Substitute values for m$\angle DEF$ and m$\angle FDE$.*
m$\angle F$ + 143 = 180	*Simplify.*
m$\angle F$ = 37°	*Subtract 143 from both sides.*

Example 3: Proving Triangles Congruent

Given: $\angle YWX$ and $\angle YWZ$ are right angles.

$\overline{YW}$ bisects $\angle XYZ$. W is the midpoint of $\overline{XZ}$. $\overline{XY} \cong \overline{YZ}$.

Prove: $\triangle XYW \cong \triangle ZYW$

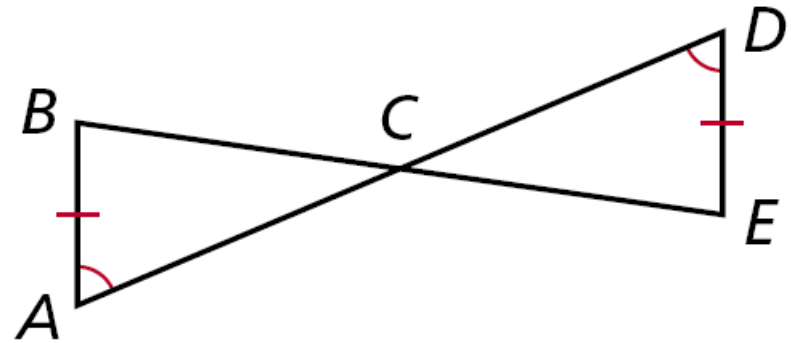| Statements	Reasons
1. ∠*YWX* and ∠*YWZ* are rt. ∠s.	**1.** Given
2. ∠*YWX* ≅ ∠*YWZ*	**2.** Rt. ∠ ≅ Thm.
3. *YW* bisects ∠*XYZ*	**3.** Given
4. ∠*XYW* ≅ ∠*ZYW*	**4.** Def. of bisector
5. *W* is mdpt. of $\overline{XZ}$	**5.** Given
6. $\overline{XW} \cong \overline{ZW}$	**6.** Def. of mdpt.
7. $\overline{YW} \cong \overline{YW}$	**7.** Reflex. Prop. of ≅
8. ∠*X* ≅ ∠*Z*	**8.** Third ∠s Thm.
9. $\overline{XY} \cong \overline{YZ}$	**9.** Given
10. Δ*XYW* ≅ Δ*ZYW*	**10.** Def. of ≅ Δ

Given: $\overline{AD}$ bisects $\overline{BE}$.

$\overline{BE}$ bisects $\overline{AD}$.

$\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$

Prove: $\triangle ABC \cong \triangle DEC$

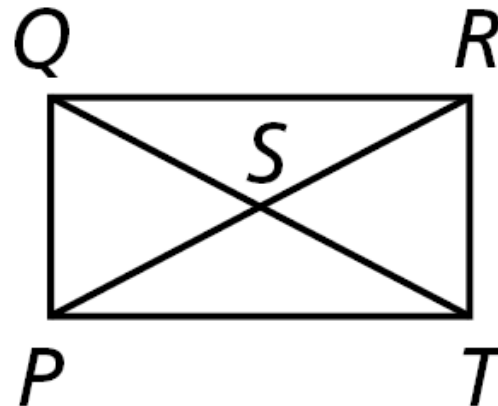| Statements	Reasons
1. $\angle A \cong \angle D$	**1.** Given
2. $\angle BCA \cong \angle DCE$	**2.** Vertical $\angle$s are $\cong$.
3. $\angle ABC \cong \angle DEC$	**3.** Third $\angle$s Thm.
4. $\overline{AB} \cong \overline{DE}$	**4.** Given
5. $\overline{AD}$ bisects $\overline{BE}$, $\overline{BE}$ bisects $\overline{AD}$	**5.** Given
6. $\overline{BC} \cong \overline{EC}$, $\overline{AC} \cong \overline{DC}$	**6.** Def. of bisector
7. $\triangle ABC \cong \triangle DEC$	**7.** Def. of $\cong$ $\triangle$s

Example 4: Engineering Application

The diagonal bars across a gate give it support. Since the angle measures and the lengths of the corresponding sides are the same, the triangles are congruent.

Given: $\overline{PR}$ and $\overline{QT}$ bisect each other.
$\angle PQS \cong \angle RTS$, $\overline{QP} \cong \overline{RT}$

Prove: $\triangle QPS \cong \triangle TRS$

Example 4 Continued

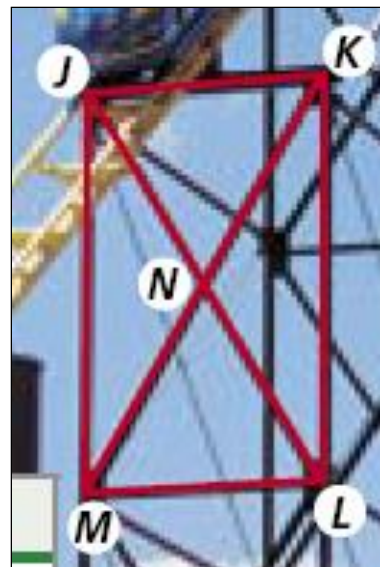| Statements	Reasons
1. $\overline{QP} \cong \overline{RT}$	**1.** Given
2. $\angle PQS \cong \angle RTS$	**2.** Given
3. $\overline{PR}$ and $\overline{QT}$ bisect each other.	**3.** Given
4. $\overline{QS} \cong \overline{TS}$, $\overline{PS} \cong \overline{RS}$	**4.** Def. of bisector
5. $\angle QSP \cong \angle TSR$	**5.** Vert. $\angle$s Thm.
6. $\angle QSP \cong \angle TRS$	**6.** Third $\angle$s Thm.
7. $\Delta QPS \cong \Delta TRS$	**7.** Def. of $\cong$ Δs

Check It Out! Example 4

Use the diagram to prove the following.

Given: $\overline{MK}$ bisects $\overline{JL}$. $\overline{JL}$ bisects $\overline{MK}$. $\overline{JK} \cong \overline{ML}$. $\overline{JK} \parallel \overline{ML}$.

Prove: $\triangle JKN \cong \triangle LMN$

Check It Out! Example 4 Continued

Statements	Reasons
1. $\overline{JK} \cong \overline{ML}$	1. Given
2. $\overline{JK} \parallel \overline{ML}$	2. Given
3. $\angle JKN \cong \angle NML$	3. Alt int. $\angle$s are $\cong$.
4. $\overline{JL}$ and $\overline{MK}$ bisect each other.	4. Given
5. $\overline{JN} \cong \overline{LN}$, $\overline{MN} \cong \overline{KN}$	5. Def. of bisector
6. $\angle KNJ \cong \angle MNL$	6. Vert. $\angle$s Thm.
7. $\angle KJN \cong \angle MLN$	7. Third $\angle$s Thm.
8. $\triangle JKN \cong \triangle LMN$	8. Def. of $\cong$ $\triangle$s

Lesson Quiz

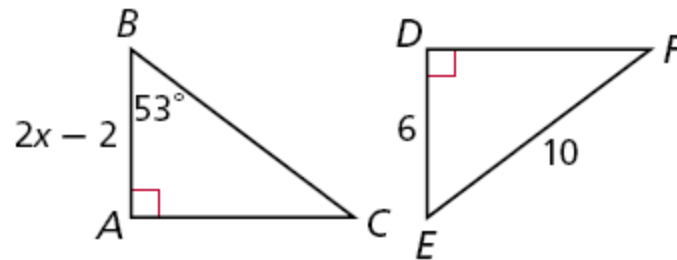**1.** $\Delta ABC \cong \Delta JKL$ and $AB = 2x + 12$. $JK = 4x - 50$. Find x and AB. 31, 74

Given that polygon *MNOP* ≅ polygon *QRST*, identify the congruent corresponding part.

2. $\overline{NO} \cong$ ___ **RS**   **3.** $\angle T \cong$ ___ **∠P**

4. **Given:** $\Delta ABC \cong \Delta DEF$

Find m$\angle F$.

Lesson Quiz

4.

Statements	Reasons
1. $\angle A \cong \angle E$	1. Given
2. C is mdpt. of BD and AE	2. Given
3. $\overline{AC} \cong \overline{EC}$; $\overline{BC} \cong \overline{DC}$	3. Def. of mdpt.
4. $\overline{AB} \cong \overline{ED}$	4. Given
5. $\angle ACB \cong \angle ECD$	5. Vert. $\angle$s Thm.
6. $\angle B \cong \angle D$	6. Third $\angle$s Thm.
7. $\triangle ABC \cong \triangle EDC$	7. Def. of $\cong$ $\triangle$s

- **HOMEWORK**

  – **Page 234 # 2 - 10, 13 – 19**